Logical Principles in Ternary Mathematics

Ruslan Pozinkevych[1*]

[1]*Faculty of Informations Technologies and Mathematics The Eastern European National University, 43021, Lutsk, Ukraine.*

Author's contributions

The sole author designed, analyzed, interpreted and prepared the manuscript.

Original Research Article

ABSTRACT

Introduction/Background: Our new research called "Logical Principles in Ternary Mathematics" is an attempt to establish connection between logical and mathematical principles governing Ternary Mathematics and address issues that appeared earlier while making truth tables for "Ternary addition" and "Ternary Multiplication" presented by the same author in "Ternary Mathematics Principles Truth Tables and Logical Operators 3 D Placement of Logical Elements Extensions of Boolean Algebra" publication.The title "Logical Principles in Ternary Mathematics" is not randomly chosen To be able to set up relations between elements in the given discipline one usually employs the basic principle of meaning-form and function In the same way we propose a logical triangle "Component","Vector","Decimal" to prove fundamental principle governing "Ternary Mathematics" presented in the given research.

Aims/Objectives: The aim of the article is to set up connection between mathematical and logical rules governing Ternary Mathematics The main postulates of the Ternary Mathematics can be demonstrated by the abstract scheme or a triangle the vertices of which are "Component","Vector","Decimal" We use a triangle diagram to prove the functionality of the chosen principle. The three components are each connected with other two and transition is possible from one to another without changing the shape of a diagram and the principle applied.

_____

Corresponding author: E-mail: galagut@protonmail.com;

Methodology: The most difficult part is to "translate" Algebra and Numeric Analysis into Mathematical Logic and vice versa Traditional methods of logic fail to do this transition therefore a new functional approach is chosen.

Results and Conclusion: As a result of this functional approach a new Ternary addition Truth Table is made The new Ternary Truth Table consists of the 3 literals (T, T̄,F) Truth Negative, False and the last column of the table is the logical sum of the two. For example: T+T=T
Unlike the old table it presents a sum of two numbers in a vector form and therefore makes it possible to use it in mathematics as well as in logic.

1. INTRODUCTION

Many of us probably have heard about syllogisms Aristotle's logic and the way we construct propositions in modern logic [1] yet a few might have thought perhaps that the choice of means or the so-called elements of a logical expression might be purely arbitrary [2]. Let me demonstrate this on an example:

We have a statement: Is it true that 4+5=7?
The statement is false but from this false statement you can obtain true assertions For example:

1) It is false that 4+5=7
2) It is not the case that 4+5=7
3) 4+5≠7

This is basically so much we can do if we rely on a binary principle applied to the above mentioned example E.g. the p denial is true when p is false and the denial of p is false when p is true (Where p is a simple statement)

Let's take a look at the 3rd example given Namely: 4+5≠7 What can we derive from the above mentioned example knowing that the statement p≡T? It implies many things for example 4+5=8 or 4+5=10 or 4+5=7+6 etc. All of these statements are false of course but based on a premise that T̄≡F we can draw a lot of wrong conclusions This is where binary principle fails to provide us with an accurate information and where we are going to step in with the ternary mathematics principle which introduces the operator T̄ as a completely independent literal the meaning of which can be described as "Some of" instead of "All of" or "None of" If we apply the above mentioned principles to the three examples given, we can see that:

1) It is false that 4+5=7
2) It is not the case that 4+5=7
3) 4+5≠7

are nothing but T̄ (truth negative) statements the meaning of which falls into neither T nor F category and in that sense are not being "governed" by the binary maths principles

2. METHODOLOGY

Given the aforementioned assumption one needs other operators than conjunction and disjunction to govern the literals Where does this assumption come from?

Then assume that we use mathematical sign to connect literals we will have something like

Let's take a look at the disjunction table.

We can see that by substituting T/ F literals by numbers we often obtain an incorrect mathematic result The reason for such inconsistency is lack of exact correlation between logic and mathematics In other words Boolean Algebra is based upon the laws of mathematics but the relations between elements

Table 1. Conjunction

A	B	$A \wedge B$
T	T	T
F	F	F
F	T	F
T	F	F

Table 2. Disjunction

A	B	$A \lor B$
T	T	T
F	F	F
F	T	T
T	F	T

Table 3. Logical multiplication

A	B	$A \land B$
1*	1	1
0*	0	0
0*	1	0
1*	0	0

Table 4. Logical addition

A	B	$A \lor B$
1+	1	1
0+	0	0
0+	1	1
1+	0	1

are logical Hence we speak about logical addition and logical multiplication which further leads to inability implementing more sophisticated mathematical apparatus to solve practical engineering technical problems utilizing principles of Boolean Algebra This has an analogy with say trying to picture somebody s portrait having only two paints black and white No matter how many colors of grey we are going to get they will still be not sufficient enough to convey the color palette of a real life image Maybe this analogy is not quite the same as the concept we are trying to build but it s quite similar Let's take a look at another example by Tony R. Kuphaldt [3],and released under the Design Science License:

Quote :"Let us begin our exploration of Boolean algebra by adding numbers together:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

The first three sums make perfect sense to anyone familiar with elementary addition.The last sum, though, is quite possibly responsible for more confusion than any other single statement in digital electronics, because it seems to run contrary to the basic principles of mathematics.

Well, it does contradict the principles of addition for real numbers, but not for Boolean numbers.

Remember that in the "world of Boolean algebra", there are only two possible values for any quantity and for any arithmetic operation: 1 or 0.

There is no such thing as "2" within the scope of Boolean values. Since the sum "1 + 1" certainly isn't 0, it must be 1 by process of elimination.

It does not matter how many or few terms we add together, either. Consider the following sums:

$$0 + 1 + 1 = 1$$
$$1 + 1 + 1 = 1$$
$$0 + 1 + 1 + 1 = 1$$
$$1 + 0 + 1 + 1 + 1 = 1$$

It clearly demonstrates us flaws in elementary or conventional math as we should say it implementing Boolean math principles We are not being critical of the latter but its high time to

turn our sights to something that employs more mathematical approach namely ternary math So what is ternary math and what rules govern it well as the name suggests it's a branch of discrete mathematics based upon utilizing three main concepts Truth, Truth Negative and False Each of them has its own specific use and is distinct from the other two In the following article we will try to prove that the same very math can be the basis for special placement of logical elements in computer CPU's microcircuits etc. [4,5]. What makes us so sure Well first of all that as we mentioned earlier three tools are better than two We can use them in various combinations and that these same very tools can be presented in a vector form and as such can help us build models in space cause as we know any vector has dual representation One is a line connecting two dots on the plane and another one points in space "cutting through the plane" or vectors in space [6].That same second representation is important for us when we want to present our vectors as a single point [7]. So far vector representation on the plane has been a set of two numbers (▫, ▫) How are we going to change that? Well we will do it by means of our "beloved" literals T T̄ F These will be our dots on the xy; yz; zx or any other plane but first we have to make it happen [8].The new relation between different number representations can be demonstrated by the following diagram: Compound-Vector-Decimal

3. MATERIALS AND METHODS

The type of methods employed in our research includes mathematical representation of logical expression and transition between Vector Algebra Numeric analysis and Mathematical Logic We used our formula to obtain a result in the form of a Ternary Addition Table as the name suggests This is not an addition of

elements in it s classic sense but rather a Logical Addition The difference between the two can be demonstrated by the following exampleT+T=T ≡ 1+1

If T=1⟹1+1≢ T+T Which means our principle is not consistent with mathematical rules Let's take a look at another example:

T̄+T̄≡(-1)+(-1)
From the course of elementary algebra, we know that (-1)+(-1)=-2 If T̄=(-1) ⟹(-1)+(-1)≠(-1) however when we apply our logical scheme the result will be exactly such:
T̄+T̄≡(-1)+(-1) ≡T̄

4. RESULTS AND DISCUSSION

All that inference clearly demonstrates our "Ternary Addition " Table:

An author made an attempt to establish a connection between logical operators (T,T̄,F) earlier
but failed due to the fact that simple mathematical addition does not apply to the logic of Ternary Maths [9].

From that perspective it is more correct to call Ternary Addition a "Logical Addition of Ternary Elements" rather than just "addition" or "Ternary Addition" Both latter definitions are wrong anyway You can compare an old attempt to describe this process by the same author:

If we substitute elements (T,T̄,F) by the numbers (1,-1,0) the entries for the second row T⊕ should result in 2 T not T and the entries for the 4th should be (-2) T≡2T̄ all of which is inconsistent from the standpoint of formal logic [10].

Table 5. Ternary addition

A	B	A ⊕ B
T	T	T
F	F	F
T̄	T̄	T̄
T	F	F
F	T	T
T̄	T	T
T̄	F	F
F	T̄	T̄
T	T̄	T̄

Table 6. (Old)

A	B	A ⊕ B
T	T	2T
F	F	F
T̄	T̄	2T̄
T	F	T
F	T	T
T̄	T	F
T̄	F	T̄
F	T̄	T̄
T	T̄	F

Failed attempt

Table 7. Ternary multiplication

A	B	A*B
T	T	T
F	F	F
T̄	T̄	T
T	F	F
F	T	F
T̄	T	T̄
T̄	F	F
F	T̄	F
T	T̄	T̄

So what did we do to change the results? According to Mathematicians namely David Hilbert and Paul Bernays [11], we can always present a single element x as a function $f(x)$

That is the basic conversion between the two We can establish a mathematical relation between all elements of the logical table to make some of their entries fall into the set of numbers $\{-1, 0, 1\}$ [12].The following principle is best demonstrated on an example

We need to establish the sum of two entries T and T The resulting entry appears to be 2T which denies logical principle of the addition: we can not speak of double Truth and if we do we will go beyond Ternary Maths cause our set of entries is 3elements : $\{-1, 0, 1\}$ and not 4,5,…etc. Let's apply our "Logical Triangle Principle"
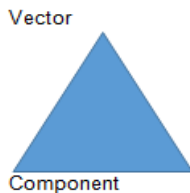
On the left we will place a sum of vectors in it's component form $\{1, 1\}$ on the right the vector form of the same sum : $v(0,1)$

Equate them $\{1, 1\} = v(0,1)$ and multiply both sides by $(0,1)$

The result is 1=v but this is the same as to say

$v(0,1) = v \Longrightarrow \{1, 1\} = 1;$

Remember that component representation of a vector is a sum of it's components (x, y)

Logical multiplication is an easier matter as the elements of the set $\{-1, 0, 1\}$ are the resulting component of the ternary multiplication table and no conversion applies

5. CONCLUSION

Our research is continuation of "Ternary Mathematics Principles Truth Tables and Logical Operators 3 D Placement of Logical Elements Extensions of Boolean Algebra" The research was published earlier by the same author We aim to make a transition between binary and ternary mathematics using logical means and mathematical principles By presenting a sum of logical elements as a vector form a new Ternary Addition principle is proposed (see Table 5.

Ternary Addition) Unlike the old table where we simply used algebraic rules for adding three numbers: -1,0,1 the new approach is to present a sum of two numbers in a vector form and by means of vector product establish a relation between logical sums and decimal numbers [13].Speaking of benefits and limitations of the proposed work one should mention that Ternary math is a sub-division of a discrete mathematics It operates similar principles and is aimed at development of the calculating machines and algorithms [14], to make counting faster and more accurate In order to be able to do 'so' we use not only calculus but vector algebra statistics and analytic geometry. In conclusion one has to point out that every mathematical operation can be successfully utilized by means of Ternary Mathematics At least basic mathematical operations such as addition and multiplication have their reflection in Ternary Math Tables and now it's the task of the branches of science to implement these principles and develop them further [15].

ACKNOWLEDGEMENT

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Duerlinger J. Sullogismos and Sullogizesqai in Aristotle's Organon. The American Journal of Philology. 1969;90(3): 320-328.
2. Lukasiewicz, J. Aristotle's syllogistic from the standpoint of modern formal logic; 1951.
3. Kuphaldt TR. Lessons In Electric Circuits, Volume I–DC. Vol Fifth Edition Open Book Project; 2006.
4. Hoshino, T. PAX Computer; High-Speed Parallel Processing and Scientific Computing. Addison-Wesley Longman Publishing Co., Inc; 1989.
5. Harris D, Harris, S. Digital design and computer architecture. Morgan Kaufmann; 2010.
6. Kravchuk, OM. Workshop on analytical geometry; 2013.
7. Dolciani MP. Modern introductory analysis. In Modern Introductory Analysis. 1967;660-660.
8. Macbeath AM. Elementary vector algebra. Oxford: Oxford University Press; 1964.
9. Pozinkevych R. Ternary Mathematics Principles Truth Tables and Logical Operators 3 D Plac ement of Logical Elements Extensions of Boolean Algebra. Asian Journal of Research in Computer Science. 2020;35-38.
10. Wittgenstein L. Ludwig Wittgenstein. Rowman & Littlefield; 2003.
11. Hilbert D, & Bernays P. Grundlagen der Mathematik II; 1974.
12. Postnikov MM. Analytic geometry. Geometry lectures; 2009.
13. Ilyin VA, Poznyak EG. Linear Algebra tr. from the Russian by Irene Aleksanova (1986). Mir Publishes Moscow; 1984.
14. Hošková Mayerová Š, Flaut C, Maturo F. Algorithms as a Basis of Modern Applied Mathematics; 2021.
15. Li F, Nicopoulos C, Richardson T, Xie Y, Narayanan V, Kandemir M.. Design and management of 3D chip multiprocessors using network-in-memory. In 33rd International Symposium on Computer Architecture (ISCA'06) 2006;130-141. IEEE.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/65652